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# Kaluza-Klein Black Holes in String Theory

Finn Larsen

*the Enrico Fermi Institute*
*University of Chicago*
*5640 S. Ellis Ave., Chicago, IL 60637, USA*
flarsen@theory.uchicago.edu

**Abstract**

Non-supersymmetric black holes carrying both electric and magnetic charge with respect to a single Kaluza-Klein gauge field have much in common with supersymmetric black holes. Angular momentum conservation and other general physics principles underlies some of their basic features. Kaluza-Klein black holes are interpreted in string theory as bound states of D6-branes and D0-branes. The microscopic theory reproduces the full nonlinear mass formula of the extremal black holes.

# 1 Introduction

Black holes are interpreted microscopically in string theory as bound states of explicitly specified constituents. It is therefore an important theoretical challenge to identify examples where the quantum bound state problem can be analyzed. The canonical example is the bound state of D1- and D5-branes where the microscopic theory is known in great detail [1]. In this and many other cases the asymptotic degeneracy of states has been determined to agree with the Bekenstein-Hawking formula for the black hole entropy. In all cases where such an agreement has been established with precision the near-horizon geometry of the black hole contains an $AdS_3$ factor; and this feature underlies the agreement [2, 3]. It remains an open problem to similarly understand black holes with less symmetric near-horizon geometries; especially black holes with no supersymmetry. The present work reports on progress in this direction, for a specific case.

Consider the original Kaluza-Klein theory in four dimensions, obtained by compactification of five-dimensional pure gravity on a circle. The field content of this theory is a $U(1)$ gauge field, a scalar field, and gravity. Stationary black holes solutions to the theory are parametrized by their electric (Q) and magnetic (P) gauge charges, as well as their mass (M) and angular momentum (J). There is a simple embedding of the system into string theory, as follows. First, add six compact dimensions, *e.g.* a Calabi-Yau three-fold, or a six-torus. Next, interpret the original Kaluza-Klein direction as the M-theory circle so that the electric Kaluza-Klein charge is identified as D0-brane charge, and the magnetic Kaluza-Klein charge similarly becomes the charge of a D6-brane, fully wrapped around the six inert dimensions. *The black hole is therefore interpreted at weak coupling as a bound state of D0-branes and D6-branes.*

In string theory it is the norm to consider black holes with many independent charges excited simultaneously. The electric and magnetic charges are thus generalized to vectors. It has been the experience that black holes with non-orthogonal charge vectors pose special difficulties: the microscopic description is less constrained [4, 5], and the corresponding classical solutions are much more complicated [6, 7, 8]. An elementary property of the Kaluza-Klein dyon considered here is that its electric

and magnetic charge vectors – being numbers – have nonvanishing inner-product, $\vec{P}\cdot\vec{Q} = PQ \neq 0$. The Kaluza-Klein black hole is therefore a simple setting where these problems can be analyzed. In fact, to the present author, this was the original motivation for considering the problem. The study of Kaluza-Klein black holes is further motivated by the "no-frill" character of the system, by fundamental string theory interest in the D0/D6 bound state [9, 10, 11, 12, 13, 14], and as a means to study black holes [15, 16, 17].

The black hole metric can be constructed explicitly for arbitrary $(M, J, Q, P)$ [18, 19, 20]. This will not be repeated here. Instead the emphasis will be on the qualitative properties of the system. The presentation is based on the article [19], except for a new result(given in section 3), the microscopic interpretation of the mass formula for the bound state .

# 2 Properties of Extreme Black Holes

For a given value of the conserved charges $Q, P, J$ there is a lowest possible value of the mass $M$ consistent with regularity. The three parameter family of solutions saturating this bound are the *extremal* black holes. The physical interpretation is that extremal black holes in some sense are in their ground states. This presentation considers only the extremal case.

## 2.1 Basic Parameters

**Black hole mass:** First, assume that the rotation is limited according to $G_4J < PQ$. In this "slow rotation" case the mass formula is:

$$2G_4M = (Q^{2/3} + P^{2/3})^{3/2} \; . \tag{1}$$

This mass formula has been known for some time, for $J = 0$ [21]; the striking point emphasized here is the *independence* of angular momentum. In other words, the system can carry angular momentum at no cost in energy. Other features of the extremal geometry *do* depend on the angular momentum, as expected.

Next, assume that $G_4J > PQ$. In this "fast rotation" case the mass formula is more complicated, the solution of a quartic equation. The mass formula now depends

on the angular momentum as well as the charges. It satisfies:

$$2G_4M > (Q^{2/3} + P^{2/3})^{3/2} \ . \tag{2}$$

The two branches of extremal black holes are joined by a two-parameter family of black holes satisfying $G_4J = PQ$. The geometry degenerates in this critical limit; for example, the black hole entropy approaches zero.

**Black hole entropy** is also sensitive to the boundary at $G_4J = PQ$. Indeed, for slow rotation $G_4J < PQ$:

$$S = 2\pi\sqrt{\frac{P^2Q^2}{G_4^2} - J^2} \ , \tag{3}$$

while for fast rotation $G_4J > PQ$:

$$S = 2\pi\sqrt{J^2 - \frac{P^2Q^2}{G_4^2}} \ . \tag{4}$$

The only change is thus the overall sign under the square root. The extremal Kerr-Newman black hole and its four-parameter generalization canonically considered in string theory [22, 23, 24], coincide for special choices of parameters with the *fast* rotation case (4). Here the main interest is the *slow* rotation case (3).

**The black hole temperature** vanishes in the extremal limit for *all* values of the angular momentum. This conforms with general expectations for extremal black holes.

## 2.2 Comparison with Supersymmetric Black Holes

The fast rotating Kaluza-Klein black holes are very similar to extremal Kerr-Newman black holes in four dimensions. A more surprising analogy is between the slowly rotating Kaluza-Klein black holes and the rotating BPS black holes in five dimensions, interpreted as excitations of D1/D5-brane bound states [25]. The striking similarities include:

**1)** In the D1/D5-case the energy of a supersymmetric ground state is related to its momentum by supersymmetry, ensuring that the black hole mass is independent of the angular momentum. The D1/D5-system therefore also has the property that it can carry angular momentum at no cost in energy.

**2)** The supersymmetric ground states of the D1/D5 system are generally charged under the R-charge of the supersymmetry algebra, which is identified with the space-time angular momentum [25]. The projection on to a given value of the R-charge restricts the available phase space, and so decreases the entropy; it vanishes when the angular momentum is so large that all states are forced to have identical projection of the angular momentum. The black hole entropy has the form:

$$S = 2\pi\sqrt{\frac{1}{4}J_3 - J^2} \ , \tag{5}$$

where $J_3$ is the unique cubic invariant of $E_{6(6)}$. This should be compared with the entropy (3), or more generally the U-duality invariant expression:

$$S = 2\pi\sqrt{\frac{1}{4}J_4 - J^2} \ , \tag{6}$$

where $J_4$ is the unique quartic invariant of $E_{7(7)}$. The similarity suggests that the Kaluza-Klein black holes are described by a supersymmetric conformal field theory with a structure similar to the one familiar from the D1/D5-system. Specifically, the angular momentum should be identified with an R-charge in such a description.

**3)** For slowly rotating Kaluza-Klein black holes the angular velocity of the horizon *vanishes* $\Omega_H = 0$ . The physical interpretation is that the angular momentum is carried by the field surrounding the black hole, rather than by its interior. The unfamiliar combination of angular momentum, but no angular velocity, occurs also for the rotating BPS black holes in five dimensions. For fast rotation, the horizon velocity remains finite in the extremal limit $\Omega_H \neq 0$, as for the Kerr black hole.

**4)** Outside the horizon of a rotating black hole there is an *ergosphere*. This is a region where observers cannot remain at rest relative to the asymptotic geometry, because the drag of the geometry force them to rotate along with the black hole. Such observers are nevertheless free to escape to infinity. An important consequence of the ergosphere is that it allows the black hole to shed rotational energy classically by *superradiance*. This effect renders *e.g.* the standard extremal Kerr black hole in four dimensions unstable. The D1/D5-system corresponds to rotating black holes in five dimensions and for these, remarkably, the ergosphere disappears in the extremal

limit. This saves the stability of the system required by supersymmetry. Interestingly, the ergosphere of the Kaluza-Klein black hole *also* disappears in the extremal limit, for slow rotation. On the other hand, for fast rotation there *is* an ergosphere; so the black hole decays classically, even though it is extremal. In a sense, the mass (2) on the large rotation branch is too large, and the black hole seeks to reach the lower bound (1) which apparently is more stable.

At this point it may appear that slowly rotating Kaluza-Klein black holes are precisely analogous to the D1/D5-system. That is far from the truth. The D1/D5 system is supersymmetric; indeed, it is the only case familiar to me where rotation is consistent with the BPS condition [26, 27]. Many of the remarkable properties discussed above follow from this fact. It is thus significant to emphasize that Kaluza-Klein black holes are *not* supersymmetric.

To see this, embed the simple Kaluza-Klein theory in a theory with at least N=2 supersymmetry. The supersymmetry algebra then implies:

$$2G_4M \geq \sqrt{Q^2+P^2} \ , \tag{7}$$

with the inequality saturated if and only if the black hole preserves a part of the supersymmetry. The mass formulae (1-2) satisfy this condition; however,they never saturate it, when both electric and magnetic charges are present. Kaluza-Klein black holes are therefore *not* supersymmetric.

Without supersymmetry, the question of stability should be considered seriously. The energy of two widely separated fragments, each carrying either the electric or the magnetic charge is:

$$2G_4M \geq Q+P \ . \tag{8}$$

This inequality is also satisfied by the mass formulae (1-2); so spontaneous fragmentation of the black hole into two parts is consistent with energy conservation.

However, in the present system it is important to consider also angular momentum conservation. The electric and magnetic fragments are charged with respect to the *same* $U(1)$ gauge field; so the total angular momentum of the final state satisfies Dirac's bound:

$$J \geq \frac{PQ}{G_4} \ . \tag{9}$$

The lower bound coincides precisely with the one classifying Kaluza-Klein black holes as having slow or fast rotation. It is interesting that the evident qualitative distinction between slow and fast rotation is related to the Dirac bound on the angular momentum: the geometry "knows" about the Dirac bound. Concretely, the bound implies that angular momentum conservation *forbids* decay of the slowly rotating black holes into two widely separated electric and magnetic fragments; but the fast rotating ones *do* decay in this way.

It is possible that the slowly rotating black hole instead decays into two widely separated *dyons*, with charge assignments $(Q_1, P_1)$ and $(Q_2, P_2)$, respectively. The Dirac bound (9) on the angular momentum of the fragments is then replaced by:

$$J \geq |P_1 Q_2 - P_2 Q_1| / G_4 \ . \tag{10}$$

For example two identical dyons can have vanishing angular momentum. There are still large classes of black holes that have no possible decays; for example non-rotating black holes with mutually prime quantized charges. In fact, standard stability arguments, using supersymmetry, are similarly subject to conditions on the quantum numbers of the state. The possible decay into two dyons is therefore consistent with the analogy between BPS states and the slowly rotating branch.

These results do not imply that the slowly rotating black holes are absolutely stable. For example, the angular momentum could be carried away by one of the decay products. An example that realizes this possibility is the Callan-Rubakov effect [28, 29]; here a charged spin-1/2 fermion interacts with a monopole, but the combined system nevertheless supports a spin-0 mode. An alternative decay channel involves a third particle carrying spin, but arbitrarily low energy; *e.g.* a graviton. Despite the existence of allowed decay channels, it is evident that the slowly rotating extremal black holes exhibit a remarkable degree of stability; in particular, it is suggestive that the most obvious decay channel is forbidden. It would be interesting to make a stronger and more precise statement on this issue.

### 2.3 Quantization Rules

Up to this point, the electric and magnetic charges have been arbitrary parameters. After embedding into quantum theory they are quantized:

$$
\begin{aligned}
Q &= 2G_4 M_0\, n_Q \ , &(11)\\
P &= 2G_4 M_6\, n_P \ , &(12)
\end{aligned}
$$

where $n_Q$ and $n_P$ are *integral*. In the D0/D6 interpretation discussed in the introduction:

$$
\begin{aligned}
M_0 &= \frac{1}{l_s g_s} \ , &(13)\\
M_6 &= \frac{V_6}{(2\pi)^6 l_s^7 g_s} \ , &(14)
\end{aligned}
$$

where the string units are defined so $l_s = \sqrt{\alpha'}$ and $V_6$ is the volume of the six compact dimensions wrapped by the $D6$-brane; as always $G_4 = \frac{1}{8}(2\pi)^6 l_s^8 g_s^2$. This gives the relation $8G_4 M_0 M_6 = 1$ (which in fact is expected from general principles). Thus:

$$
\frac{2PQ}{G_4} = n_Q n_P \ . \tag{15}
$$

As a check on normalizations note that, after this quantization condition is taken into account, the lower bound in (9) quantizes the angular momentum as a half-integer. A related point is that the entropy (3-4) simplifies. After the quantization condition is taken into account it is expressed in terms of pure numbers, *i.e.* the moduli cancel out. This is promising for a connection to microscopic ideas.

## 3 The Microscopic Description

The analogy with BPS black holes suggests that it is possible to describe Kaluza-Klein black holes precisely in the underlying string theory. As discussed in the introduction, the microscopic interpretation of the Kaluza-Klein black hole is a bound state of $k = n_Q$ D0-branes and $N = n_P$ D6-branes. The theory on the D6-branes is a field theory in 6+1 dimensions. The field content is the same as maximally supersymmetric Yang-Mills theory with $SU(N)$ gauge group. D0-branes are described in this theory as excitations with third Chern-class equal to the number of D0-branes, and vanishing

first and second Chern-classes. Assuming that the compact dimensions span a six-torus, it is simple to construct examples of this kind using time-independent field strengths of the form:

$$F_{12} = f\mu_1 \ ; \qquad F_{34} = f\mu_2 \ ; \qquad F_{56} = f\mu_3 \ , \tag{16}$$

where the $SU(N)$ matrices $\mu_i$ satisfy:

$$\mu_i^2 = I \ ; \qquad \mu_1\mu_2\mu_3 = I \ ; \qquad \mathrm{Tr}\mu_i = 0 \ ; \qquad \mathrm{Tr}\mu_i\mu_j = 0 \ . \tag{17}$$

The first and second Chern-classes vanish because the trace of $F$, and also of $F \wedge F$, vanish along all cycles. The third Chern-class – and so the number of D0-branes – is:

$$k = \frac{1}{6(2\pi)^3} \int \mathrm{Tr} F \wedge F \wedge F = \frac{1}{(2\pi)^3} N V_6 f^3 \ . \tag{18}$$

It is convenient to use (11-14) and rewrite this relation as $(2\pi)^3 l_s^6 f^3 = Q/P$.

The D6-brane wraps a small compact manifold, so it is legitimate to ignore higher derivatives in the action. The interactions of the theory are therefore given by the Born-Infeld Lagrangean. For static configurations the corresponding mass functional is:

$$M = T_6 \int \mathrm{Tr} \sqrt{\det\left(1 + 2\pi l_s^2 F\right)} \ , \tag{19}$$

where $T_6$ is the tension of the $D6$-brane, *i.e.* its mass density. For the explicit configurations given above the mass becomes:

$$M = T_6 V_6 N \left(1 + (2\pi)^2 l_s^4 f^2\right)^{3/2} = \left(P^{2/3} + Q^{2/3}\right)^{3/2} / (2G_4) \ . \tag{20}$$

This is precisely the mass formula (1). A similar computation was presented in [9], for charges $N = k = 4$ and moduli chosen such that $P = Q$. The generalization given here shows that a full functional dependence is reproduced by the microscopic considerations.

# 4 Discussion

The D0/D6-interpretation of the system is valid at weak coupling, *i.e.* when the ambient spacetime is mildly curved. In contrast, the black hole description applies at strong coupling. The agreement between the mass formulae obtained in the two

mutually exclusive regimes therefore suggests a duality. Indeed, although Kaluza-Klein theory does *not* admit an $SL(2,\mathbb{Z})$ duality group, it preserves a $\mathbb{Z}_2$ subgroup interchanging electric and magnetic charges, as well as weak and strong coupling. The mass formula is therefore not necessarily invariant under extrapolation from weak to strong coupling; but the two regimes are related by a discrete symmetry.

It is simple to construct explicit microscopic configurations of the form (16-17); indeed, there are numerous ways to do so. Moreover, if the *ansatz* is relaxed, there are additional possibilities. The system therefore has considerable microscopic degeneracy which is presumably related to the black hole entropy. However, a precise confirmation of this idea has not yet been achieved.

Another open problem concerns the world-volume interpretation of the angular momentum. As discussed after (6), the black hole entropy formula suggests a relation to the R-charge of a superconformal algebra; or at least some $U(1)$ current in a 2D conformal field theory. Unfortunately it seems difficult to identify a specific current with the required properties.

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